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"Drift" Waves in a Partly Ionized Plasma

by

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Abstract

A dispersion relation for the "drift" waves is obtained. Terms taking into account momentum exchange between charged particles and neutrals extend earlier results to the case of partly ionized plasmas. The dispersion relation is solved numerically for the case of a Q-machine plasma in which a neutral gas is introduced. The effects of the neutrals on the maximum growth rate and on the corresponding wave-length parallel to the magnetic field are discussed.

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Introduction

Low-frequency ion waves propagating perpendicularly to the magnetic field in an inhomogeneous low-β (ratio of material to magnetic pressure) plasma have been investigated theoretically by N. D'Angelo[1]. A dispersion relation was obtained on the basis of the macroscopic equations, resistivity being left out of account. It was shown that a wave propagating perpendicularly to both density gradient and magnetic field, with a phase velocity equal to the ion pressure-gradient drift velocity, may exist. F. F. Chen[2] has treated the problem including finite resistivity and finite values of the wave number parallel to the magnetic field. His most important result is that for very small wave numbers parallel to the magnetic field some of the modes can be excited by the pressure gradient.

The purpose of this report is to examine how the different modes, found earlier, behave if the plasma is not fully ionized. Two questions are of special interest: (1) Do the neutrals affect the growth rate of the unstable modes and, if so, in what way? (2) Does the parallel wave number for a given excitation shift towards higher values? The latter question is interesting from an experimental point of view because, in a fully ionized plasma, the most excited unstable modes have very large parallel wave-lengths, which are difficult to measure precisely. If the presence of a large number of neutrals reduces those wave-lengths considerably, the experimental verification of the excitation caused by the pressure gradient may be easier.

Equations

The analysis is based on the Boltzmann moment equations:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \underline{v}_i) = 0 \tag{1}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \underline{v}_e) = 0 \tag{2}$$

$$n m_i \left(\frac{\partial \underline{v}_i}{\partial t} + \underline{v}_i \cdot \nabla \underline{v}_i\right) + \varkappa T_i \nabla n + qn(\nabla \varphi - \underline{v}_i \times \underline{B}) = \underline{P}_{ie} + \underline{P}_{in} \tag{3}$$

$$\varkappa T_e \nabla n - qn(\nabla \varphi - \underline{v}_e \times \underline{B}) = \underline{P}_{ei} + \underline{P}_{en} \; . \tag{4}$$

The subscripts i, e and n denote ions, electrons and neutrals respectively; the $\underline{v}$'s are the fluid velocities, $T_{i,e}$ the temperatures and φ the electric potential; the P's take the collisions between the different particles into account; e.g., P_{ie} is the momentum transferred per unit volume and time from electrons to ions. We have neglected the electron inertia term, and each plasma component is taken to be isothermal. We also neglect viscosity and changes in the magnetic field produced by the plasma motion (low-β approximation). Finally we assume charge neutrality ($n_i \approx n_e = n$). Since we are dealing with low-frequency oscillations, this assumption should be reasonably fulfilled. For simplicity the calculations are carried out in a Cartesian frame of reference. We take the B-field in the z-direction and the density gradient in the x-direction.

The collision terms may be written as

$$\underline{P}_{ie} = - \underline{P}_{ei} = - \frac{m_e m_i}{m_e + m_i} \, n^2 \, \sigma_{ei} \, v_{ei} \, (\underline{v}_i - \underline{v}_e) \quad , \tag{5}$$

i.e. the momentum transferred is proportional to the reduced mass, the cross section for momentum transfer σ_{ei}, the relative velocity v_{ei} of two colliding particles, and the difference between the two fluid velocities. In analogy with what is done in ref. 2 and for the same reasons as given there, only the z-components of the terms representing momentum transfer in collisions are retained. With $m_e \ll m_i$ and $m_i \approx m_n = m$, we can write the collision terms as

$$\begin{aligned}
\underline{P}_{ie} &= - m_e \, n^2 \sigma_{ei} \, v_{ei} \, (v_{iz} - v_{ez}) \, \hat{z} \\
\underline{P}_{in} &= - \tfrac{1}{2} \, m \, n_n \, n \, \sigma_{ni} \, v_{ni} \, (v_{iz} - v_{nz}) \, \hat{z} \\
\underline{P}_{en} &= - m_e \, n_n \, n \, \sigma_{ne} \, v_{ne} \, (v_{ez} - v_{nz}) \, \hat{z} \quad ,
\end{aligned} \tag{6}$$

where $\hat{z}$ is a unit vector in the z-direction.

Equilibrium

To perform a linearization analysis of the equations (1) to (4) we must first choose a zero-order state. A possible zero-order state will be one in which the electric field vanishes, the velocities have only y-components, the neutral density is constant, and the plasma density depends only on the x-co-ordinate. With these requirements eqs. (1) to (4) are satisfied if the y-components of the ion and electron velocities are the diamagnetic drift velocities

$$v_{ioy} = -\frac{c_i^2}{\omega_{ci}}\lambda(x) ; \quad v_{eoy} = \beta\frac{c_i^2}{\omega_{ci}}\lambda(x) , \tag{7}$$

where

$$\omega_{ci} = qB/m ; \quad c_i^2 = \kappa T_i/m ; \quad \lambda(x) = -n_o^{-1}\partial n_o/\partial x ; \quad \beta = T_e/T_i .$$

In addition, the magnetic field is uniform in space and constant in time.

Perturbations

To make the analysis reasonably simple we assume that the neutral particles form a fixed, cold background, i.e. we do not take into account a possible wave motion of the neutrals. Therefore, by linearizing eqs. (1) to (4) we obtain

$$\frac{\partial \underline{v}_{i1}}{\partial t} + \underline{v}_{i1}\cdot\nabla\,\underline{v}_{io} + \underline{v}_{io}\cdot\nabla\,\underline{v}_{i1} + \frac{c_i^2}{n_o}\nabla n_1 + \frac{q}{m}\nabla\varphi_1 - \frac{q}{m}\,\underline{v}_{i1}\times\underline{B}$$

$$-\frac{q}{m}\frac{n_1}{n_o}\,\underline{v}_{io}\times\underline{B} - \alpha n_o\,\sigma_{ei}\,v_{ei}\,(v_{ez1} - v_{iz1})\,\hat{z} + \frac{1}{2}\,n_n\,\sigma_{ni}\,v_{ni}\,v_{iz1}\,\hat{z} = 0$$

$$\beta\frac{c_i^2}{n_o}\nabla n_1 - \frac{q}{m}\nabla\varphi_1 + \frac{q}{m}\,\underline{v}_{e1}\times\underline{B} + \frac{q}{m}\frac{n_1}{n_o}\,\underline{v}_{eo}\times\underline{B} \tag{8}$$

$$+\,\alpha n_o\,\sigma_{ei}\,v_{ei}\,(v_{ez1} - v_{iz1})\,\hat{z} + \alpha n_n\,\sigma_{ne}\,v_{ne}\,v_{ez1}\,\hat{z} = 0$$

$$\frac{\partial n_1}{\partial t} + n_o\,\nabla\cdot\underline{v}_{i1} + \nabla n_o\cdot\underline{v}_{i1} + \nabla n_1\cdot\underline{v}_{io} = 0$$

$$\frac{\partial n_1}{\partial t} + n_o\,\nabla\cdot\underline{v}_{e1} + \nabla n_o\cdot\underline{v}_{e1} + \nabla n_1\cdot\underline{v}_{eo} = 0 ,$$

where

$$\alpha = \frac{m_e}{m_i} .$$

We want to analyse waves propagating in the y-z plane, i.e. in the plane perpendicular to the density gradient. Therefore we take the perturbed quantities proportional to $\exp[i(k_y y + k_z z - \omega t)]$. We take the first-order velocities and the potential to be independent of x. Since, for the case of electrostatic waves, $n_1/n_o = q\varphi_1/kT$, we have thereby chosen the first-order density to have the same x-dependence as the zero-order density. We write

$$
\begin{aligned}
n_1 &= n_1^*(x)\, e^{i(k_y y + k_z z - \omega t)} \\
\underline{v}_{i,el} &= \underline{v}_{i,el}^*\, e^{i(k_y y + k_z z - \omega t)} \qquad\qquad (9)\\
\varphi_1 &= \varphi_1^*\, e^{i(k_y y + k_z z - \omega t)} .
\end{aligned}
$$

Now eqs. (8) lead to the following system of eight equations with the perturbed quantities as unknowns:

$$
\begin{aligned}
-i\chi\, v_{i1x}^* + v_{i1y}^* &= 0 \\
i c_i^2 \gamma_y \nu + (1+A) v_{i1x}^* - i\chi v_{i1y}^* + i\frac{q}{m}\gamma_y \varphi_1^* &= 0 \\
i c_i^2 \gamma_z \nu + (\Gamma_i - i\chi) v_{i1z}^* + i\frac{q}{m}\gamma_z \varphi_1^* - \Gamma_{ei} v_{e1z}^* &= 0 \qquad (10)\\
v_{e1y}^* &= 0 \\
i\beta c_i^2 \gamma_y \nu - v_{e1x}^* - i\frac{q}{m}\gamma_y \varphi_1^* &= 0 \\
i\beta c_i^2 \gamma_z \nu - \Gamma_{ei} v_{i1z}^* - i\frac{q}{m}\gamma_z \varphi_1^* + \Gamma_e v_{e1z}^* &= 0 \\
-i\chi\nu - \delta v_{i1x}^* + i\gamma_y v_{i1y}^* + i\gamma_z v_{i1z}^* &= 0 , \\
-i\left[\chi + \gamma_y(1+\beta)c_i^2\delta\right] - \delta v_{i1x}^* + i\gamma_y v_{e1y}^* + i\gamma_z v_{e1z}^* &= 0
\end{aligned}
$$

where

$$\chi = \Omega - \gamma_y v_{ioy} ; \quad \Omega = \frac{\omega}{\omega_{ci}} ; \quad \gamma_y = \frac{k_y}{\omega_{ci}} ; \quad \gamma_z = \frac{k_z}{\omega_{ci}}$$

$$\delta = \frac{\lambda}{\omega_{ci}} ; \quad \nu = \frac{n_1^*}{n_o} ; \quad A = \frac{1}{\omega_{ci}} \frac{\partial v_{ioy}}{\partial x} ;$$

$$\Gamma_{ei} = \frac{\alpha n_o \sigma_{ei} v_{ei}}{\omega_{ci}} ;$$

$$\Gamma_{ni} = \frac{n_n \sigma_{ni} v_{ni}}{2 \omega_{ci}} ;$$

$$\Gamma_{ne} = \frac{\alpha n_n \sigma_{ne} v_{ne}}{\omega_{ci}} ;$$

$$\Gamma_i = \Gamma_{ei} + \Gamma_{ni} ; \quad \Gamma_e = \Gamma_{ei} + \Gamma_{ne} ; \quad \Gamma_n = \Gamma_{ni} + \Gamma_{ne} .$$

From the determinant of eq. (13) we obtain the dispersion relation for the "drift" waves

$$\begin{aligned} &\chi^4[\delta\gamma_y \Gamma_e + i\gamma_z^2] + \chi^3[i\delta\gamma_y(\Gamma_e\Gamma_i - \Gamma_{ei}^2) - \gamma_z^2 \Gamma_n - \gamma_y^2 \Gamma_e] \\ &- \chi^2[A\delta\gamma_y \Gamma_e + (1+\beta) c_i^2 \delta\gamma_y\gamma_z^2 \Gamma_{ei} + i\gamma_y^2 (\Gamma_e\Gamma_i - \Gamma_{ei}^2) \\ &+ i(1+\beta) c_i^2 \gamma_y^2 \gamma_z^2 + i\gamma_z^2(1+A) + i(1+\beta) c_i^2 \gamma_z^4] \\ &+ \chi [i(1+\beta) c_i^2\gamma_y\gamma_z^2\delta + (1+\beta) c_i^2\gamma_y^2\gamma_z^2 \Gamma_i + (1+A)\gamma_z^2\Gamma_n - iA\delta\gamma_y(\Gamma_e\Gamma_i - \Gamma_{ei}^2) \\ &-(1+\beta)\delta c_i^2\gamma_y \gamma_z^2\Gamma_i + (1+\beta)\delta c_i^2(1+A)\gamma_y\gamma_z^2 \Gamma_{ei} + i(1+\beta)(1+A)c_i^2\gamma_z^4 = 0 . \end{aligned} \tag{11}$$

The dispersion relation (11) is reduced to that of ref. 1 (taken in the limit $m_e = 0$) when γ_z, Γ_{ei}, Γ_{ne}, Γ_{ni}, and A are put equal to zero. The condition $A = 0$ means that the zero-order density profile is taken to be exponential, i.e. λ is constant. Similarly, the dispersion relation in ref. 2 is obtained from eq. (11) by taking finite values of γ_z and Γ_{ei} into account.

Numerical Computation

The dispersion relation is solved by a standard numerical procedure. For this purpose we have chosen a caesium plasma immersed in a magnetic field of 5 kG. We restrict ourselves to the case of a constant λ ($A = 0$). From experimental observations in Q-machines it is known that the e-folding length of the density profile is generally between 5 and 10 times the Larmor radius[3]. For the temperatures we want to consider we may take $1/\lambda = 1$ cm. A typical value for the diameter of the plasma column in Q-machines is 3 cm. Therefore, for an "m = 1" mode we may take $\gamma_y = 1.8 \cdot 10^{-6}$ sec/cm.

The resistivity term, Γ_{ei}, is specified by comparing the ion-electron momentum exchange term (5) with

$$\underline{\Gamma}_{ei} = - q^2 n^2 \eta \, (\underline{v}_i - \underline{v}_e) \, ,$$

where the resistivity, η, is given by[4]

$$\eta = 6.53 \cdot 10^3 \, \frac{\ln\Lambda}{T_e^{3/2}} \quad \text{ohm cm.}$$

We choose a plasma density of 10^{11} cm^{-3}, and we want to consider electron temperatures between $2.5 \cdot 10^3$ and $25 \cdot 10^3$ °K. Therefore we may take $\ln\Lambda$ constant and equal to 7.5.

For the collision terms concerning the interaction between the charged particles and the neutral gas we have to specify the relative velocities and the cross sections for momentum transfer. Since we consider the neutral particles as a cold background, the velocities v_{en} and v_{in} are essentially the electron and the ion thermal velocity respectively. In the derivation of the dispersion relation (11) we took the case of equal masses for the ions and the neutrals. Experimentally this applies to the case of a caesium plasma and xenon as neutral gas. For the cross sections we take the values for caesium-caesium[5]; they are $\sigma_{ne} = 3 \cdot 10^{-15}$ cm^2 and $\sigma_{ni} = 7 \cdot 10^{-15}$ cm^2. In the electron temperature range we are going to consider, the value for the electron-neutral cross section should be reasonably correct. We want to consider ion temperatures from room temperature up to $2.5 \cdot 10^3$ °K. The cross section mentioned for the ion-neutral collisions is not correct for such low temperatures. Thus we see that neither v_{in} nor σ_{ni} is determined accurately. However, it turns out that the ion-neutral collisions do not influence the essential points in the discussion very much. We might even take our results to be valid in the case of caesium-xenon although the value of σ_{ni} used includes the effect of resonant charge exchange.

Results and Discussion

(a) The Case of $T_i = T_e$

Fig. 1 shows a typical result of the numerical computation. The phase velocity in the y-direction normalized to the ion diamagnetic drift velocity is plotted for different values of θ^2. θ is defined as the ratio between the parallel and the perpendicular component of the wave vector. This ratio, for small values, is essentially the angle between the wave vector and the y-direction. Positive imaginary parts of $\omega/k_\perp$ correspond to growing modes. Fig. 1 shows the case of a fully ionized plasma. The curves therefore coincide with those given in ref. 2 except for an unimportant, damped mode which we have lost by neglecting the electron inertia term. λ is here taken negative as in ref. 2.

Fig. 2 shows the solution for the case of a partly ionized plasma. The ratio n_n/n_o is taken to be 10^4, and we now want to inspect this particular case. The mode labelled (4), which has a phase velocity almost equal to the ion drift velocity for all values of θ considered here, is damped by the neutrals. This is not important physically because this mode is not excited either in the case of a fully ionized plasma. The imaginary part of this mode is not shown in fig. 1, but it is negative, although very small. The additional damping is due to the ion-neutral collisions. Therefore, according to waht we noted in the last section the actual figures for this imaginary part cannot be trusted entirely.

By comparing figs. 1 and 2 we also see that the entire family of curves is shifted towards higher values of θ^2, but the shape of the curves remains unchanged, i.e. the maximum growth rate for the modes which are growing for $n_n = 0$ is not changed by the neutrals. The mode (3) is a high-frequency mode (Mc/sec range). Therefore we fix our attention on the low-frequency mode (1), which is believed[2)] to be the mode seen in Q-machine experiments[6)]. The shift in the curves means that the parallel component of the wave-length, $\lambda_\parallel$, for which the maximum growth rate occurs, becomes shorter in the presence of neutrals. In fig. 3 we have plotted θ for the maximum growth of mode (1) for different values of the ratio n_n/n_o. We see that $k_\parallel$ increases with increasing n_n when the ratio n_n/n_o is above 10^3. Physical arguments show that this effect is reasonable.

The electric field of the wave parallel to the magnetic field is equal to the product of the current, J, and the resistivity, $\eta_{\parallel}$. On the assumption that J is independent of n_n, this product increases with n_n because the neutral particles will increase the resistivity along the B-lines. If the potential difference between peak and trough along the plasma column is independent of n_n, the product of the E-field and $\lambda_{\parallel}$ is independent of n_n. Therefore we may expect a smaller wave-length corresponding to a higher resistivity.

We noticed that the effects mentioned here only appear when the ratio n_n/n_o is above 10^3. This is due to the fact that the ratio between the Coulomb cross sections and the cross sections for ion-neutral and electron-neutral collisions is of the order of 10^3.

(b) The Case of $T_i < T_e$

In this section we want to analyse the case of different ion and electron temperatures. We consider this case because of the well-known fact that the introduction of a noble gas into the plasma column of a Q-device changes the ion temperature considerably, whereas the electrons remain at the temperature of the hot end plate. To simulate this case we take our parameter β to be equal to 10. One could also obtain this, or even a greater, value of β by heating the electrons by microwave power. In figs. 4a and b we have plotted the low-frequency, excited mode of the dispersion relation (11) for $\beta = 10$ and $n_n/n_o = 10^4$; in fig. 4a $T_i = 2500^oK$, and in fig. 4b $T_e = 2500^oK$. This means that fig. 4a corresponds to the case of heating the electrons and fig. 4b to the case of cooling the ions. The fact that we have fixed the ion temperature at 2500^oK in fig. 4a although the degree of ionization is only 10^{-4} may seem unrealistic for Q-devices. This choice makes comparison between different figures easier, and the more realistic case of $T_i = 250^oK$ and $\beta = 100$ would give analogous results. In both figures we have normalized $\omega/k_\perp$ to the value of v_{ioy} that corresponds to $T_i = 2500^oK$.

Concerning the maximum growth rate we notice by comparison between figs. 4 and 2 that its absolute value has been increased by the increase in T_e and decreased by the decrease in T_i (if we had plotted the growth rate versus the actual value of v_{ioy}, we should have found an increase in both cases). In fig. 5 we have plotted the maximum growth rate and the corresponding phase velocity for different values of T_e ($T_i = 2500^oK$). The curves in the figure are independent of the ratio n_n/n_o as in the case of $T_e = T_i$.

The results obtained here may seem contradictory to what was observed in the experiment of Blau et al.[7]. They excited low-frequency oscillations by introducing a noble gas into the plasma column in a Q-device. The amplitude was found to have maxima at the positions of the largest density gradients, i.e. one would expect the oscillations to be "drift" waves. We noticed above that the maximum growth rate decreases for decreasing T_i. It is, however, difficult to apply our theory to the experiment of ref. 7, which was performed with the machine operating single-ended with a negative sheath. The negative sheath limits the parallel wave-length, but if the cold plate is floating, an ion sheath will form there[8]. Therefore it is impossible to figure out what values of $\lambda_{\parallel}$ we have to consider. It seems that much experimental work remains to be done in order to clarify these observations.

We shall now fix our attention to the ratio of $k_{\parallel}/k_{\perp}$ for which the "drift"-mode (1) attains its maximum growth rate, and try to find out what Q-machine conditions favour the precise measurement of the parallel wave-length of this "drift"-mode. The curve in fig. 3 is almost unchanged when the ion temperature decreases, as will be seen later, i.e. $k_{\parallel}/k_{\perp}$ is nearly independent of the ion-neutral collisions. The perpendicular wave-length with which we are concerned here is of the order of 10 cm; the lowest degrees of ionization with which one can run Q-devices at present are 10^{-3} - 10^{-4}. This means (fig. 3) that the parallel wave-length is almost unchanged from the case of a fully ionized plasma ($\lambda_{\parallel} \sim$ 50 m). We shall now look at the case in which the electron temperature is increased. If $n_n = 0$, this will lead to greater values of $\lambda_{\parallel}$ because of decreasing resistivity, but for n_n/n_o sufficiently large the contribution to the resistivity from electron-neutral collisions dominates the effect from ion-electron collisions. This can be seen from fig. 6, where we have plotted $k_{\parallel}/k_{\perp}$ as a function of n_n/n_o for two different values of the electron temperature ($T_e = 2.5 \cdot 10^3\,^oK$ and $T_e = 25 \cdot 10^3\,^oK$). In both cases $\beta = 10$, but, as mentioned above, the curves are not very sensitive to changes in β. By comparing the curve in fig. 3 ($\beta = 1$) with the curve for the case of the same T_e in fig. 6 we see that the decrease in T_i (factor of 10) causes a small increase in $\lambda_{\parallel}$ (factor of 1.3).

The two curves in fig. 6 intersect at $n_n/n_o \sim 50$, i.e. for our particular choice of parameters the contributions to the resistivity parallel to the magnetic field from ion-electron and electron-neutral collisions are about equal for this value of n_n/n_o. For higher values of this ratio the parallel wave-length for maximum excitation of the "drift" mode (1) is smaller in the case of higher T_e because of electron-neutral collisions. Taking again as lowest possible degrees of ionization 10^{-3} - 10^{-4}, we find that for our particular choice of the diameter of the plasma column the parallel wave-length for maximum excitation is of the order of 10 m.

Conclusions

If we restrict ourselves to the low-frequency, excited solution of the dispersion relation (11), the following conclusions may be drawn. The maximum growth rate is independent of the ratio n_n/n_o provided that T_i and T_e can be kept constant. In Q-machine plasmas this growth rate increases with the electron temperature, but is a decreasing function of the ion temperature. The parallel wave-length for which maximum excitation occurs decreases when the ratio n_n/n_o increases if the ratio is larger than a minimum value depending on the electron temperature. Finally, the parallel wave-length for maximum growth decreases when T_e increases for n_n/n_o sufficiently large.

Our results may be related to experiments in Q-machines. In addition they may be relevant in the study of the ionosphere; they are of course not restricted to the values of n_n/n_o considered here.

We have neglected the finite Larmour radius effects. Eqs. (1) to (4) have been solved numerically, both viscous damping and FLR-effects[9] being included. As in the case of a fully ionized plasma[10], the inclusion of FLR-effects moves the real part of the "drift"-mode (1) from one to zero for $k_\theta = 0$. Therefore the curve in fig. 5 giving the phase velocity for maximum excitation cannot be entirely trusted as far as actual figures are concerned. In the case of a partly ionized plasma our treatment may be more correct than the treatment including the FLR-effects. If the ion motion is almost entirely dominated by collisions, i. e. if the ion-neutral collision frequency, ν_{in}, is of the same order as or larger than the ion cyclotron frequency, then our treatment seems to be more correct. For a neutral density of 10^{15} cm^{-3}, $\nu_{in} = n_n \sigma_{in} v_{in}$ is of the same order as ω_{ci}. The results are thus mainly concerned with this collision-dominated regime.

Acknowledgements

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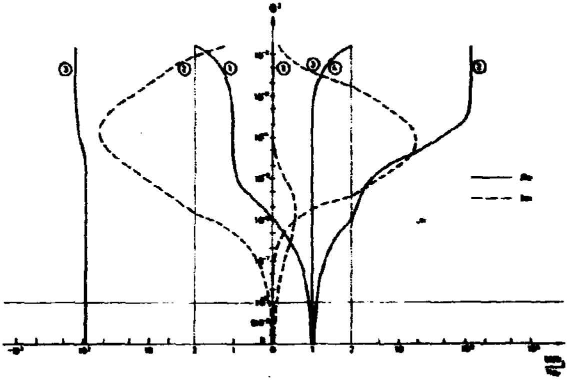

Fig. 1. Real and imaginary parts of ω/k_z normalized to the zero-order ion drift velocity plotted for different values of θ^2, θ being the ratio between k_z and $k_\perp$; $n_o = 10^{11}$ cm^{-3}; $n_n/n_o = 0$; $\lambda = -1$ cm^{-1}; $T_e = T_i = 2500^0K$; $B = 5$ kG; diameter of plasma column 3 cm; "m = 1" mode.

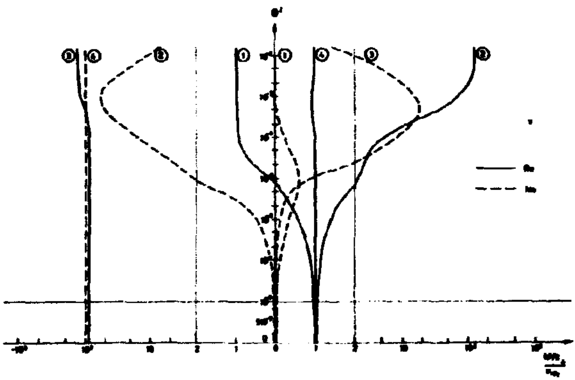

Fig. 2. The solutions of the dispersion relation for the same parameters as in fig. 1 except that $n_n/n_o = 10^4$.

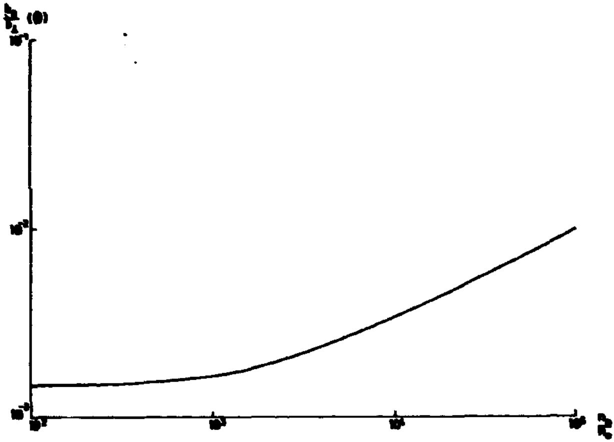

Fig. 3. The ratio $k_{\parallel}/k_{\perp}$ for maximum growth of mode 1 versus the ratio n_n/n_o; $n_o = 10^{11}$ cm^{-3}; $T_e = T_i = 2500^{\circ}$K.

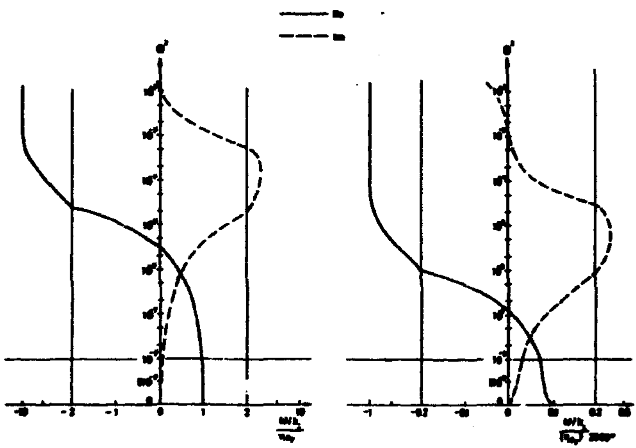

Fig. 4. The low-frequency, excited mode plotted for two different cases:

(a) $T_i = 2500^{\circ}$K; $T_e/T_i = 10$

(b) $T_e = 2500^{\circ}$K; $T_e/T_i = 10$

$n_o = 10^{11}$ cm^{-3}; $n_n/n_o = 10^4$; $\lambda = -1$ cm^{-1}; $\omega/k_{\perp}$ is normalized to the same velocity in the two cases.

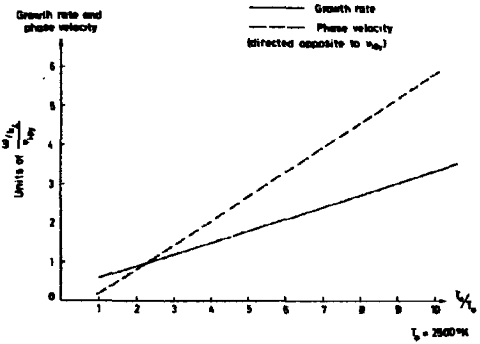

Fig. 5. Maximum growth rate and corresponding frequency of mode 1 versus T_e; T_i = 2500°K.

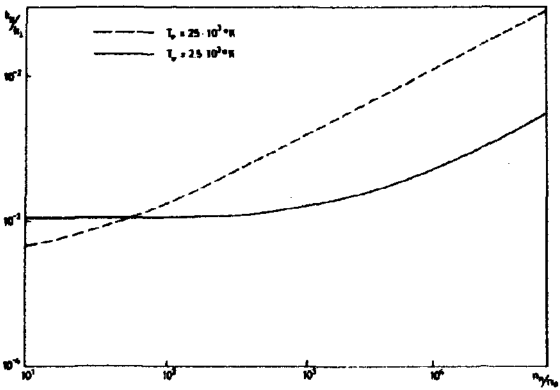

Fig. 6. The ratio $k_z/k_\perp$ for maximum growth of mode 1 versus the ratio n_n/n_o in the cases (a) T_e = 2500°K and (b) T_e = 25000°K (T_e/T_i = 10 in both cases); $n_n = 10^{11}$ cm^{-3}.